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The Nature and Efficacy of Methods of Attack on Reasoning Problems

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Benjamin Burack

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I. STATEMENT OF THE PROBLEM*

THE LITERATURE on reasoning reveals a variety of methods used or usable by subjects during problem-solving. Although some of the methods are reported as having been experimentally observed in psychological studies, other methods appear to be personal reflections on how problems could be solved. Whereas the former could actually be either efficacious, indifferent, or detrimental to successful solution, the latter are implicitly presumed to be advantageous. Nine of these "methods of attack" have been selected for investigation in this experiment, in an attempt to determine their use and efficacy for a variety of problems.

There are three questions with which we shall deal:

1. How do individuals differ in the methods they use to solve problems?

2. To what extent are these methods used in solving problems?

3. To what extent are different methods efficacious?

The nine methods selected for investigation are described below.

1. *Clear formulation of the problem.* Dewey early emphasized the importance of awareness of the specific problem. In his first edition of *How We Think,* he discussed as the first 2 of his 5 steps in reflective thinking: (*a*) a felt difficulty, and (*b*) its location and definition.

2. *Preliminary survey of all aspects of the presented material.* The Gestalt School has stressed the role of survey of the whole field—the situation as a whole. And Durkin (6) found that solution was facilitated by initial familiarity with all the parts of her construction-puzzles.

3. *Analysis into major variables.* Seashore (17, p. 554) writes, "In any large and difficult problem, one of the first steps is to analyze out its main variables, which may then be attacked one at a time, in simpler fashion." Here the problem-solver might ask himself this question: "What are the principal factors which determine or influence the solution of this problem?" It is recognized that the problem-solver may utilize variables not helpful for solution.

4. *Locating a crucial aspect of the problem:* "Loci" (Ruger), "Direction" (Maier), "Analysis" (Heidbreder). The subject might question: "What part of the problem seems to merit special attention as the central factor relative to solution?" Ruger found his subjects becoming aware of the "locus" of difficulty. Maier demonstrated the importance of the "way the problem is attacked." However, Maier also warns against "habitual behavior" or "persistence" as *false direction,* wrong attack, a side-track away from correct direction leading to solution. And Heidbreder suggested that the function of analysis is to reduce the supply of stimulus-content by singling out possibly-significant features. As with major variables, the problem-solver may pursue aspects "crucial" to him, but actually not helpful to solution.

5. *Application of past experience.* The significance of applying past experience ("transfer") is a commonplace (Barlow, Chant, Kinnamon, Lindworsky, Lyon). Drever (5) concluded that the application is not necessarily conscious.

6. *Varied trials.* By definition, repeti-

* In accordance with new editorial policy, the review of the literature on reasoning, forming the background of this inquiry, is omitted in this monograph, but is a part of the original dissertation, available on interlibrary loan from the Northwestern University Library, Evanston, Illinois.

tion of the same kind of trial could not be successful unless it was correct the first time (also note the previously mentioned warning of Maier against persistence in an unsuccessful direction). Variability has been established as one of the prime factors conducive to solution (Bain, Durkin, Lloyd Morgan, Thorndike, J. C. Peterson, *et al.*).

7. *Control.* In experimentation, control (variation, isolation, repetition) is foundational. In solving a problem, the question can be put: "How can I control each factor involved so as to determine its effect and relative influence?"

8. *Elimination of sources of error.* The literature reveals scarce mention of the role of this factor. What sources of error (for instance, unsteadiness in a delicate mechanical puzzle) may be eliminated or controlled so as to facilitate solution of the problem? As with other methods (see 3 and 4, above), the problem-solver may eliminate factors which are not actually sources of error.

9. *Visualization.* Chen, Comstock, Eidens, Pear, Storring, Wilhelm, and many others have demonstrated the role of imagery in reflective thinking. In behavioristic terms imagery may be considered as the use of implicit trials.

Even at the outset, it appeared that some overlapping exists among certain of these methods, for example "analysis into major variables" and "locating a crucial aspect of the problem."

The subjects were 25 students enrolled in an advanced undergraduate course in psychology. It was recognized that any conclusions concerning efficacy, and especially extent of use, of specific methods would be based on a highly select group of subjects, and that the experiment should be considered only as an exploratory study opening the way to systematic study of problem-solving in the general population.

Three problems were chosen, not as representative of typical human problems, but because their simplicity would permit some objective analysis relatively uncomplicated by the non-intellectual (e.g., emotional) factors often affecting daily problem-solving. The three problems, involving induction, deduction, and geometrical-figure analysis, are described in the next section.

Our problem, then, was to determine the use and efficacy of nine known "methods of attack" on different problems.

II. THE PROBLEM-MATERIALS

A. The Induction Problem

1. *Construction*

THE INDUCTIVE PROBLEM, largely concept-formation (devised by Professor Thurstone), is presented in full, below. Five groups of letters are presented, with four groups always containing something in common; the subject is to *detect the group not containing the feature common to the other four groups.* Exclusive of the practice-problems, 19 such problems were given.

LETTER-GROUPING

In the five groups of letters below, notice that four of the groups have something in common. The one which is not like the other has been marked.

YLNPJ	XBPZR	YNTFH	YRGSW	YHDLY

Four of the groups begin with the letter Y. The one which does not begin with Y is marked.

Look at the next problem.

AAXVP	FNTBB	HQTQX	JDSWR	MECTE

Four of the groups have a repeated letter. The first, second, third, and fifth groups have the letters AA, BB, QQ, and EE respectively. The fourth group does not have a repeated letter. It is unlike the other groups so it is marked.

Work the next two problems. Put a mark under the group which is different in each case.

ABCDE	XYZPS	EFGHI	CXVTN	PJKLM
EDCBA	VWXYZ	IHGFE	JKLMN	SBTGK

In the first problem four of the groups have letters occurring in alphabetical order. You should have put a mark under the fourth group.

In the second problem four of the groups are in alphabetical order, left to right or right to left. The fifth group should be marked.

Look at the next two problems. Try to find which one should be marked.

BDHQX	SDAPZ	GXMOT	LEZTQ	NXUFS
KCUPE	FZEGO	UMBXA	APIQT	LNUKH

In the first problem four of the groups each contain a vowel. The first group should be marked.

In the second problem four of the groups have two vowels. Group five should have been marked.

Now work the next two problems. In each case put a mark under the group which is different.

JPIDX	KPHMA	LPZSU	MPLJE	NPRWO
PZDDD	SSSPB	JPPPN	HNNNX	BQBFB

In the first problem four of the groups have the vowel in the last place. The first group should be marked.

In each group of the second problem there is a letter occurring three times. In four of the groups the three identical letters are together. The last group should be marked.

Work the problems which follow.

1. VBCDF	WQRSH	XIJKF	YLMNF	ZABCF
2. ELMNE	FABCF	GOPQG	HZTMH	IVWXI
3. LGBGM	TBKAK	ZDNEN	FRLLC	VPSPW
4. XVTQZ	BFDHJ	HDFBJ	JBFDH	HJFBD
5. BCAHV	BCOHV	DZELM	DKILM	DTCLM
6. FCRCT	CJCCL	CPCSC	CCRCQ	CKCFC
7. BWBRB	BBCBS	RBMBB	BNBBV	XBBZB
8. BQHTM	AUIEO	ZDKRN	TMPZB	HXKQN
9. EFGHI	EDCBA	UTSRQ	ALRXI	OPQRS
10. CEDGH	FAGHB	DKCES	LMCED	MPCEK
11. CFAGG	PESRP	VROVF	NOMSN	PROTS
12. ONMLK	LKJIH	UTSRQ	GKMOQ	HGFED
13. ALMNA	PBCDP	EVRHE	UJKLU	IDVLI
14. CCPPR	MSSMR	TRRTR	JNNJR	WWDDR
15. BSQVM	TDPDX	GHGRG	FPSFR	MPMBM
16. ACTRM	ABSPQ	CDTGH	PQBXY	MNCST
17. JKLMV	BPQRS	KNWXY	BCDQT	CXTRF
18. FGHLM	CDFGH	JKMNO	STVWX	PQSTU
19. VMSAT	DGMSB	SPLOM	QTMSC	MSDHP

2. *Administration*

This group of problems was administered individually to 20 of the 25 subjects (5 students reported familiarity with this induction-test; their results were excluded). As per directions on the form, the subject first worked all of the practice-problems. Starting with the last set of (five) practice-problems, the subject was questioned, as soon as he finished each one, as to why he had selected the one group of letters which he had marked. His answer was recorded as either "correct," or in full if he had marked the right group but gave an incorrect reason or if he had marked the wrong group and gave some reason for so doing. After he was finished, the following four questions were asked so as to detect evidence of use of methods of attack:

1. What are you supposed to do in these problems? (To detect "Clear formulation of the problem.")

2. Try to list all the ways you can remember in which four groups had something in common lacking in the fifth group. (To detect "Analysis into major variables.")

3. Have you ever had any previous experience with a similar kind of problem? (If yes:) Were you able to apply this past experience here? How? (To detect "Application of past experience.")

4. Do you remember anything else concerning the procedure you used which you believe was helpful in solving these problems? (To detect any other methods of attack.)

Since the three specific questions (exclusive of the last general question) tested for only three methods, we relied upon careful examination of the subject's worksheet to detect evidence of other methods used. It was also recognized that each question was only *one* way of testing for use of any particular method of attack, that subjects unable to state use of a method may use the method, perhaps unconsciously, and that other questions and ways of detecting a specific method could be devised; these limitations are discussed below, under "Discussion" and "Summary and Conclusions." As with the other problems administered, the subject was allowed as much time as he wished for these induction-problems.

B. The Deduction Problem

1. *Construction*

The deductive problem (selected as one of 15 items in one of Professor Thurstone's Deductive Reasoning Tests) was presented in printed form, as follows:

(*a*) Captain Watts and his son James have been found shot—the father in the chest and the son in the back. (*b*) Both clearly died instantaneously. (*c*) A gun fired close to the person—as, for example, when a man shoots himself—will blacken and even burn the skin or clothes; fired from a greater distance it will leave no such mark. (*d*) The two bodies were found near the middle of a large hall used as a rifle range. (*e*) Its floor is covered with damp sand which shows every footprint distinctly. (*f*) Inside the room there are two pairs of footprints only. (*g*) A third man standing just outside the door or window could aim at any part of the room, but the pavement outside would show no footmarks. (*h*) Under Captain Watts' body was found a gun; no such weapon was found near James. (*i*) In each case the coat, where the bullet entered, was blackened with gunpowder, and the cloth a little singed. (*j*) Captain Watts was devoted to his son and would have died sooner than harm him purposely; hence it is impossible to suppose that he killed him deliberately, even in self-defense. (*k*) But some think that James secretly disliked his father and hoped to inherit his fortune at his death.

a. To what was Captain Watts' death due? Murder? Accident? Suicide?

b. To what was James' death due? Murder? Accident? Suicide?

2. *Administration*

This problem was administered individually to all 25 students. Then the following questions were asked:

1. After reading the problem-questions at the end of the story, did you re-read the entire paragraph, keeping in mind constantly the three possibilities of *murder, accident, suicide*? (Since it early became evident that all subjects understood the problem set to them here, we used this question as probing for the possibility of extreme fixation of the answer-possibilities, or extremely vivid "Formulation of the problem.")

2. How many times did you read the whole paragraph? (To detect careful "Preliminary survey of all aspects of the presented material.")

3. Did you conclude that certain bits of given information were important or relevant, either directly or indirectly, for solving this problem? If so, consult the paragraph, and list all such points. (To detect "Locating a crucial aspect of the problem.")

4. Have you ever had any previous experience with a similar kind of problem? If so, were you able to apply this past experience here? How? (To detect "Application of past experience.")

5. Did you try to apply the possibility of *each* of the three causes—*murder, accident, suicide*—to the death of Captain Watts—and then to the death of James—to determine whether the cause could have been *murder*, could have been *accident*, could have been *suicide*? (To detect "Varied trials.")

6. If you re-read the paragraph, did you single out *each sentence* to try to determine whether any of the various sentences made impossible one or two of the three possible causes? (To detect "Control.")

7. Consult the paragraph for the following questions:

a. Was there any information in the paragraph which makes the following sentence *not* helpful in solving the deaths: "A third man standing just outside the door or window could aim at any part of the room but the pavement outside would show no footmarks."

b. Was there any information which indicated whether *Captain Watts* or *James* died first?

c. Was there any information which made *murder* improbable for James' death?

d. Was there any information which made *suicide* improbable for James' death? (To detect "Elimination of sources of error.")

8. Did you find yourself visualizing the various persons and events? (To detect "Visualization.")

9. Do you remember anything else concerning the procedure you used? (To detect any other methods of attack.)

C. The Geometrical-Figure Problem

1. *Construction*

The geometrical-figure problem (suggested by Lindley's (10) study) is presented in Figure 1. As with many other similar figure-problems, this figure is to be drawn without going over any line already drawn and without taking the pencil off the paper. Starting at any point except either of the two inner diagonal points makes solution impossible. Fourteen trials were possible on the sheet.

2. *Administration*

This problem was administered individually to 24 of the 25 students (one subject accidentally started at the correct

starting-point; his paper was excluded). The subject was allowed 14 trials. If he solved the puzzle on or before the fourteenth trial, he was asked to continue, and was stopped after making 3 successes in a row. The experimenter kept a record of the starting and finishing points for each trial. Then the following questions were asked:

1. Did you assume at the outset that the directions required that you start at some specific corner? (Since pre-experimental investigation had revealed that many subjects started at the upper left-hand corner, we included this question to detect any invalid "Formulation of the problem.")

2. Before beginning the puzzle, did you make a preliminary survey of all aspects of the pictured puzzle, or did you start drawing after a glance at the puzzle? (To detect "Preliminary survey of all aspects of presented material.")

3. Did you conclude at any time that some special part of the puzzle was best to start at? If so, what part? (To detect "Locating a crucial aspect of the problem.")

4. Have you ever had any previous experience with a similar kind of problem? If so, were you able to apply this past experience here? How? (To detect "Application of past experience.")

5. Did you discover certain moves which made solution impossible, or moves which you decided to avoid? If so, what moves or combination of moves? (To detect "Elimination of sources of error.")

6. Did you ever visualize the various possible next-moves before even tracing out these moves on the paper? If so, about how many moves ahead did you visualize on the average? (To detect "Visualization.")

7. Do you remember anything else concerning the procedure you used which you believe was helpful in solving this puzzle? (To detect any other methods used.)

III. RESULTS

BECAUSE it would break continuity to list the findings on extent of use for each of three problems and then have to return to the problems for data on efficacy, we present here the complete results for each of the three problems. In the "Discussion" of this paper, we shall return to our two main questions: *use* and *efficacy* of various methods of attack.

A. The Induction Problem

Table 1 shows the distribution of scores. The range is small probably because the items, selected from a speed-test, were taken in unlimited time and were solved, in most cases, by almost all the subjects.

TABLE 1

Scores	No. of Subjects
19	4
18	6
17	4
16	3
15	1
14	2
	$N = 20$

Mean = 17.15
Stand. Dev. = 1.53

1. *The Methods of Attack Used*

As contrasted with the other two problems to be discussed, the induction problem revealed only three distinct methods of attack.

(*a*) ***Clear formulation of the problem.*** The initial instructions read: "In the five groups of letters below, notice that four of the groups have something in common. The one which is not like the others has been marked.

YLNPJ XBPZR YNTFH YRGSW YHDLY

Four of the groups begin with the letter Y. The one which does not begin with Y is marked."

The other examples given (8 altogether) were similar. For each problem, four of the five groups have something in common lacking in the remaining group. It must be noted that the fifth group must be not merely different, but different in the sense of lacking a characteristic common to the other groups; *each* group was probably different from the other four in the sense of containing, say, some letter absent from the other four groups, but this kind of difference was not the required kind. The failure to make the distinction between merely being different and being different in the sense of *lacking a characteristic common to four groups* became evident in the results. A majority of the subjects formulated the problem simply as: Find the group which is different. We have considered this as an "incomplete" formulation.

Only 40% of the subjects correctly formulated the problem, verbally. However, incomplete *verbal* formulation did not seem to involve *actual* incomplete formulation during the working of the problems, as can be observed by the fact that even the lowest score was: 14 correct out of a possible 19, or 74% correct (chance, for 5 choices, would permit 20% correct); it seems axiomatic that successful solution of this kind of problem would require understanding the task. Apparently the nature of the task was evident in spite of the incomplete verbal formulation. This difficulty of mistaking *inability to state a method* for *failure to detect or use a method* was anticipated

(see Administration of the Induction Problem, above), and continued to obscure the use of methods in all three problems, as further described in this paper. Incomplete formulation led, however (in a few cases), to the kind of error which might be expected: selecting a group which was merely different. The three examples found are given below:

SUBJECT RLD: BCAHV BCOHV DZELM DKILM DTCLM

The subject's reason: "The others do not have a letter Z." Note that to carry out his incorrect formulation of the problem at this point the subject would be obliged to mark the first group as alone containing an A, the second group as alone containing an O, the fourth group as alone containing a K, and the last group as alone containing a T. Selecting the third group because it alone contained the letter Z indicates incomplete (if not incorrect) formulation. This subject's verbal formulation was: "Pick the one group that is different from the other four groups."

SUBJECT MN: CCPPR MSSMR TRRTR JNNJR WWDDR

Although the group marked is the correct one, the subject's reason is wrong, close however to borderline correctness: "This group has the same letter three times." The correct reason is: The other four groups each have one and only one letter R, or (another reason): The other four groups each have three different letters. This subject's verbal formulation was: "To find the group which is different."

SUBJECT MEM: BSQVM TDPDX GHGRG FPSFR MPMBM

The subject's reason: "The other four groups do not have any letter repeated three times." As a matter of fact her statement is false, for the third group has the letter G three times. But even if her statement were true, her reason indicates the same reversal of the problem-task: she finds a characteristic absent from four groups when the instructions require her to find a characteristic present in four groups. Her verbal formulation was: "Underline the one that is different in any respect from the other four groups."

(*b*) *Preliminary survey of all aspects of the presented material.* The particular nature of this problem required every subject to survey all 5 groups of letters; this factor was found in 100% of the subjects.

(*c*) *Analysis into major variables.* Our question asked the subject to list all the ways in which four groups had something in common lacking in the fifth group. Analysis of all the problems (8 practice-problems and 19 test-problems) gave the following list as including every possible reason for the 27 problems that each subject encountered:

Four groups have some common letter (or 2, 3, 4, or 5 letters).
Four groups have only one of a letter common to all five groups.
Four groups have 2 given letters adjacent.
Four groups have some common letter in a constant position (say, fourth in the group of letters).
Four groups have some vowel (or 2 vowels).
Four groups have some vowel in a constant position.
Four groups have five consonants.
Four groups have three different letters.
Four groups have a repeated letter (or a letter three times).
Four groups have a repeated letter adjacent (or separated).
Four groups have a repeated letter separated by one letter.
Four groups have an alphabetical sequence (of 2, 3, 4, or 5 letters).
Four groups have 2 alphabetical sequences of 2 and 3 letters, respectively.

Analysis of these thirteen ways in which four groups had something in common further revealed 8 major variables basic to all such problems:

Letters.
Vowels.
Consonants.

Position of a letter.

Alphabetical sequence.

Adjacency or separation of 2 given letters.

Quantity of a given letter.

Quantity of different letters within a group.

We credited a subject for each such variable stated or indicated in his list of "ways in which four groups had something in common."

Each of the 8 variables was acknowledged by one or more subjects. Some subjects were more thorough in formulating a variable; for example, some remembered and stated that alphabetical sequence could be either forward or backward.

The distribution of the number of major variables reported is shown in Table 2.

Although other methods of attack may have been employed by the subjects, no other methods revealed themselves as having been used on this problem. Experience with this problem leads us to suggest the following reasons for the fact that only three methods appeared for this problem:

1. Locating a crucial aspect of the problem: Here, it seems (to the writer) that this particular problem does not contain a "crucial aspect" (further explained in the "Discussion").

2. Application of past experience: The 20 subjects had reported, before the experiment, that they had not previously worked problems of this kind.

3. Varied trials: Variation could mean several things in this problem. (*a*) If it be considered as referring to the subject's questioning each group's lack of a characteristic common to the other four groups, then the small amount of stimulus-material probably permitted every subject to try all five groups, time being unlimited. (*b*) If variation be considered as referring to trying various generalizations (of characteristics possibly common to four of the five groups), then this "method" becomes synonymous with "Analysis into major variables," reported, above, for this problem.

4. Control: Same as for "Varied trials."

5. Elimination of sources of error: As with (1) above, the writer doubts that this factor is adequately meaningful for this problem.

6. Visualization: The stimulus-material itself is visual in nature.

The full implications of the apparent absence of certain methods on certain problems will be more completely treated in the "Discussion."

The range of the total number of methods used per subject was from two to three, with the mean at 2.4 methods per subject.

A question of special interest was: Why did our subjects make the various errors we found on their work-records? We have analyzed the 19 errors made, in relation to the reasons the subjects gave in each case for marking the group of letters they selected:

TABLE 2

Major Variables Reported	Number of Subjects
7	4
6	7
5	6
4	3
	$N = 20$

Mean = 5.6 variables
Stand. Dev. = .97

	Errors	
Perceptual (visual) errors:	12	(63%)
Wrongly formulated problem-task:	3	(16%)
Miscellaneous (hunches, etc):	4	(21%)
Total:	19	

63% of the errors were perceptual mistakes. We list here some examples:

VBCDF WQRSH XIJKF YLMNF ZABCF

SUBJECT PHK: "Because the other groups have the three middle letters in alphabetical sequence."
But so does the one she marked.

LGBGM TBKAK ZDNEN FRLIC VPSPW

SUBJECT KLJ: "Because the other four groups have one letter repeated."
But so does the one she marked.

FGHLM CDFGH JKMNO STVWX PQSTU

SUBJECT JAH: "Because this group is the only one with a vowel."
But note the last group.

16% (3 errors) were the result of wrongly formulated problem-task and have been already listed and discussed above under "Clear formulation of the problem."

2. *The Efficacy of the Methods Used*

Following are the correlations between the methods used and the number of problems worked correctly.

(*a*) *Clear formulation of the problem.* Clear verbal formulation correlated (biserial correlation) −.10 with problems worked correctly, so individual differences in verbal formulation in this experiment do not seem to be related to successful solution. Apparently subjects understood the task in spite of inability to state the task clearly and correctly.

(*b*) *Preliminary survey of all aspects of presented material.* All subjects used this factor, so no comparative evidence was available concerning its efficacy.

(*c*) *Analysis into major variables.* The number of major variables reported correlated (product-moment correlation) +.51 (P.E. = .11) with problems worked correctly, so detection of major variables seems to be moderately correlated (positively) with successful solution.

B. The Deduction Problem

Following is the distribution of scores on the problem concerning the deaths of Captain Watts and his son James.

	Students	
Correct cause for both deaths:	15	(60%)
Correct cause for one death:	10	(40%)
Correct cause for neither death:	0	

Given unlimited time, the majority, 60%, solved the deaths of both Captain Watts and his son James. Of the minority, 40%, some ascertained the correct cause of Captain Watts' death, and some ascertained the correct cause of James' death.

1. *The Methods of Attack Used*

All except one method (Analysis into major variables) were found.

(*a*) *Clear formulation of the problem.* Apparently the nature of the task was self-evident, for all of the subjects stated the problem-task correctly. Two subjects seemed to have formulated the problem more vividly by extreme fixation of the answer-possibilities: these two subjects reread the entire paragraph while keeping in mind the three possible causes (murder, accident, and suicide).

(*b*) *Preliminary survey of all aspects of the presented material.* Table 3 shows the distribution of the number of times the whole clues-paragraph was read. Thus, about half of the subjects did not read through the entire paragraph even a second time. All of the subjects, however, referred back to individual parts of the paragraph.

(*c*) *Locating a crucial aspect of the*

TABLE 3

No. of Readings	No. of Subjects
4	2
3	3
2	8
1	12
	$N=25$

Mean = 1.8 readings

problem. The question asking the subjects to list each item of information important to solution involved every item except two: sentence (g) concerning a third man, and sentence (k) concerning James' dislike of his father. Of the 9 important points, Table 4 shows the distribution of the number of points reported by the subjects. As an average, the subjects reported more than ⅔ of the important items of information.

TABLE 4

No. of Important Points Reported	No. of Subjects
9	1
8	5
7	10
6	5
5	3
4	0
3	1
	$N=25$

Mean = 6.7 points
Stand. Dev. = 1.26

(*d*) *Application of past experience.* Only 3 subjects reported such application. The remarks are quoted:

SUBJECT MEM: "Yes, like the fact that a man can't shoot himself in the back."
SUBJECT SEC: "Yes, I have built up a habit of considering every statement."
SUBJECT RMS: "Yes, the value of reviewing the facts by reading them again."

(*e*) *Varied trials.* On the question "Did you try to deliberately apply the possibility of *each* of the three causes—*murder, accident, suicide*—to the death of Captain Watts—and then to the death of James—to determine whether the cause could have been *murder,* could have been *accident,* could have been *suicide,*" we found that 28% (7 subjects) answered "Yes" and 72% (18 subjects) answered "No." Thus only about ¼ of the subjects felt that they had tried to consider the possibility of each of the three causes for each man's death.

(*f*) *Control.* On the question[1] "If you re-read the paragraph, did you single out each sentence to try to determine whether any of the various sentences made *impossible* one or two of the three possible causes," we found that 44% (11 subjects) answered "Yes" and 56% (14 subjects) answered "No." Thus a little less than half the subjects reported the use of this particular method of control.

(*g*) *Elimination of sources of error.* The four questions testing for ability to detect and eliminate sources of error involved a total of 8 items in the paragraph. Table 5 shows the distribution of the number of points correctly reported by the subjects. Thus, of a maximum possible of 8 items, the subjects, as an average, detected half the items.

(*h*) *Visualization.* 84% (21 subjects) reported visualization of the persons and events in the story, while 16% (4 subjects) reported absence of visualization.

The general question (concerning use of other methods) brought forth remarks by only three subjects. The first subject, on noticing that she was to solve a "detective story," said that she had a good

[1] Here, as in the other questions probing for use of methods, the particular question asked is, of course, only one way of searching for methods, in this case for control of individual clues.

TABLE 5

No. of Points Reported	No. of Subjects
6	3
5	7
4	8
3	5
2	1
1	0
0	1
	$N=25$

Mean = 4.1 points
Stand. Dev. = 1.32

imagination. Her solution would be satisfactory except for ignoring the gunpowder-burns on the back of James' coat: "Captain Watts accidentally shot himself in the chest. The bullet went through his body and hit James, standing behind him, in the back."

The remarks of the other two subjects (both of whom solved the problem correctly) suggest, again, that subjects may not be aware of using methods and may not be aware of how they actually solve a problem: "Just reading and restating the points seemed to suggest solution," and "Reading the material suggested various possible solutions which I checked on."

The only method not discovered was "Analysis into major variables." For this particular problem this method could refer to the three ways in which each man could have died (murder, accident, suicide); then the method would be synonymous with "Varied trials," reported on this problem. On the other hand, if the important clues are considered as major variables, then this method would be synonymous with "Locating a crucial aspect," reported on this problem.

The range of the total number of methods used per subject was from 3 to 7, with the mean at 4.7 methods per subject. Of course this average conceals the qualitative differences in the use of a given method by different subjects and also the quantitative difference in the extent to which a method was used.

2. *The Efficacy of the Methods Used*

The following tables and bi-serial correlations show the relationship between the use of a method and the success or failure in solving the two deaths.

(*a*) *Clear formulation of the problem.* All subjects seemed to formulate the problem clearly, so no measure of efficacy was possible. It might be noted that the two subjects who formulated the problem quite vividly (by fixating the three answer-possibilities while rereading the paragraph) both solved the problem.

(*b*) *Preliminary survey of all aspects of the presented material.* The number of times the subject read the clues-paragraph correlated +.22 with successful solution, so complete re-survey seems slightly related to successful solution.

(*c*) *Locating a crucial aspect of the problem.* The number of important points detected and reported correlated +.39 (P.E. = .15) with successful solution, so detection of this kind of crucial aspect seems slightly related to successful solution.

(*d*) *Application of past experience.* Only 3 subjects reported the direct application of past experience (2 solved the problem), so no measure of the efficacy of past experience was feasible.

(*e*) *Varied trials.* In interpreting the following percentages (in Tables 6, 7, and 8), it must be noted that 60% of the entire group solved the problem. Therefore the percentage of users of the method "Varied trials" must be compared

TABLE 6

	Solved (by 60% of the subjects)	Failed (by 40% of the subjects)
7 subjects who tested three possible causes	5 (71%)	2 (29%)
18 subjects not testing three possible causes	10 (56%)	8 (44%)

with *60%*, not with 50% (as would be the case if equal numbers solved and failed the problem). Of the 7 subjects who tested each of the three possible causes, 71% solved the problem, 11% more than might be expected, so (our measure of) variation seems slightly related to successful solution.

TABLE 7

	Solved (by 60% of the subjects)	Failed (by 40% of the subjects)
11 subjects who tested the effect of isolated items	9 (82%)	2 (18%)
14 subjects not testing the effect of isolated items	6 (43%)	8 (57%)

(*f*) *Control.* Of the 11 subjects who isolated items and tested their effects, 82% solved the problem, 22% more than might be expected, so this measure of control seems somewhat related to successful solution.

(*g*) *Elimination of sources of error.* The number of items detected concerning sources of error correlated +.60 (P.E. = .12) with successful solution, so our measure of detection and elimination of sources of error seems to be moderately and positively correlated with successful solution.

(*h*) *Visualization.* Of the 21 subjects who reported visualization of the persons and events in the story, 57% solved

TABLE 8

	Solved (by 60% of the subjects)	Failed (by 40% of the subjects)
21 subjects who reported visualization	12 (57%)	9 (43%)
4 subjects not reporting visualization	3 (75%)	1 (25%)

the problem, 3% less than might be expected, so visualization does not seem related to successful solution.

The total number of methods used correlated +.48 (P.E. =.14) with successful solution, and thus was moderately and positively correlated with success in this problem.

Thus, four methods seem somewhat related to successful solution of this particular problem:

Locating a crucial aspect
Varied trials
Control
Elimination of sources of error

C. The Geometrical-Figure Problem

Table 9 shows the distribution of the number of trials required to solve the problem, and includes the 6 subjects who failed to solve the problem in the 14 trials allowed.

1. *The Methods of Attack Used*

This tracing-puzzle is so constructed that, by starting at either one of the two end-points of the inner diagonal, solution is easy. But starting at any other of the 14 remaining points makes solution impossible. Of course it is possible that a subject will happen to start at one of the two correct starting-points; one subject did start thus and was surprised at his early solution, and his paper was excluded from our results since no infor-

TABLE 9

Trials	No. of Subjects
3	4
4	4
5	1
6	2
7	1
9	1
12	1
13	2
14	2
Failed	6
	$N = 24$

Median = 8 trials

mation concerning work-methods was available. Of the nine methods of attack, only "Application of past experience" was not reported.

(*a*) *Clear formulation of the problem.* All of the subjects seemed to have formulated the problem clearly; apparently the task in this puzzle is self-evident, from the nature of the instructions employed. Although many subjects started (and tended to continue starting) at the upper left-hand point, they had not assumed that the directions required that they start at this corner; possibly the deeply ingrained reading-habit of starting at upper-left may underlie this phenomenon (Lindley, 10). One special aspect of the problem-task, however, was found to be unclearly formulated by a few subjects, who asked whether they might erase their last line drawn; the directions specified ". . . do not erase or change what you have drawn. . ."

(*b*) *Preliminary survey of all aspects of the presented material.* 38% of the subjects reported that they had studied the puzzle before beginning to draw. In one case a subject pondered for half a minute before beginning the first trial. On this particular problem, the time spent in preliminary survey was very short; subjects tended to gain familiarity with the problem through overt trial-and-error rather than through close inspection or implicit (imagined) trial-and-error.

(*c*) *Analysis into major variables.* Since starting at either of the two correct starting-points makes solution rather easy, there is only one major variable: correct starting-point. However, since it is possible to go into a blind-alley even after starting correctly, the avoidance of blind-alleys may be considered as a minor variable. The subjects reporting the major variable all achieved the same general idea: starting at the center ("it seemed best to start at the center and work around"). However, the minor variable was not so specifically formulated. Although most of the subjects who reported it simply warned against blind-alleys, others were more specific

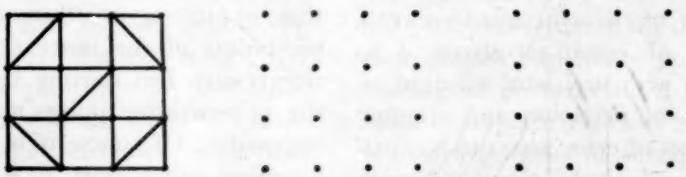

FIGURE 1. Geometrical-figure problem.

Draw this figure, without going over any line already drawn and without taking the pencil off the paper. Fill in as many lines as you can. If you are unable to complete the figure, *do not erase or change* what you have drawn, but start over again on a new figure. *Continue* until you draw a complete figure successfully.

("Leave an open space near a partially drawn section to get back in later to complete the section." And another: "Don't start at any of the 4 outside corners."). 17% of the subjects reported the major variable. 38% reported the minor variable (blind-alleys). And none reported both.

(d) Locating a crucial aspect of the problem. Some subjects hit upon a *"false crucial aspect";* they selected one point for starting and felt that solution depended only upon the care exercised after starting. Another false "crucial aspect" or "direction" consisted of detecting patterns: These subjects noticed the large octagon-pattern and tried to make it first, which necessitated starting at a point which was (unknown to the subject) detrimental to solution. Finally, some subjects conceived the idea of starting in the center, "to work around"; since two of the four center-points were correct starting-points, this was a partial localization of the problem, and the subjects soon solved the puzzle. 50% of the subjects reported some "crucial" aspect (including false ones).

(e) Application of past experience. Although there were a few subjects who had worked a similar kind of puzzle in the past, none reported the application of any specific principle carried over from previous related experiences. Again, however, the criterion (verbalization) may have obscured minor aspects of carryover. Theoretically, the one great application that might have been made would be the notion of *trying different starting points.* Inspection reveals, however, that many "similar" puzzles are so constructed that, *by starting at any point and exercising care,* one can solve the puzzle; if this is true of the geometrical-figure puzzles previously tried by the subjects, then no positive carryover of the major variable (correct starting point) in our puzzle could be expected. On the contrary, we should expect a disregard of the starting-point as a negative transfer to this problem. As reported below, under "Varied trials" and "Control," many of the subjects started consecutive trials from the same point, thus confirming the possibility of negative transfer from previous puzzles in which starting-point may have been unimportant. This finding also suggests that the efficacy of methods of attack may not be general but conditional, depending upon the degree to which any given method makes clear the crucial major variable or variables basic to the problem at hand. A similar finding is reported under efficacy of the factors of "Varied trials" and "Control."

(f) Varied trials, (g) Control. As experimentally defined, "control" ("variation, isolation, repetition") has been broken down into two factors for this problem: (f) Varied trials ("variation") and (g) Control ("isolation, repetition").

Our only expedient measure of "Varied trials" was: the number of different starting-points used, divided by the number of trials required for solution. Thus, the highest possible variability would be 100% for a subject who varied his starting-point on each new trial. The lowest possible variability would be 7% (really representing no variability), for a subject who tried only one starting-point in 14 trials ($1 \div 14$). It should be noted, however, that this measure of variability refers to *variability of starting-point* and that there remains the complex variability displayed *after* starting. This latter variability, important no doubt to the subject, did not permit measurement; thus we were un-

able to establish a general measure of variability.

Our measure of "Control" was: Total number of trials in which the same starting-point was used two or more times consecutively, divided by the number of trials. Thus, a subject who started at one point for three trials, then at another point for two consecutive trials, and then at another point for, say, five successive trials, has demonstrated 100% control (3 + 2 +5, divided by 10), in the sense that each new starting-point has been tried out for 2 or more trials before being abandoned. A subject who varied his starting-point on each new trial would demonstrate zero control (and, it should be noted, a variability of 100%). Thus, by definition for our limited means of measurement, high variability tended to go with low control (for this problem). However, since variation is an important aspect of control in the latter's full meaning and use, we have here considered our two measures as two aspects of the same principal factor.

The results showed "Variability" ranging from 14% (only 2 starting-points in 14 trials) to 100% (different starting-point on each trial), with the mean at 56%. "Control" ranged from 0% (changed starting-point on each trial) to 100% (each new starting-point tried for 2 or more consecutive trials), with the mean at 51%. Two interesting examples of introspections are quoted, as indicating personal differences among subjects in carrying out empirically the sensed awareness of the efficacy of variation:

SUBJECT ALG (Variability 36%; Control 43%): On starting her sixth trial at a new point, said, "Variety is the spice of life, so I'll start here." On starting her last possible trial (14th), she said, "Maybe if I start in the middle, something might happen." She started at a correct point and solved the puzzle.

SUBJECT RMS (Variability 14%; Control 93%): This subject began her second trial at a different point, but continued starting at this point right through the next 13 trials, unsuccessfully as regards the solution. While working on her seventh trial, she said, "Maybe if I start at a different place, I'd make it." But she started her eighth trial at the same point again. On starting her tenth trial at still the same point, she said again, "Maybe I don't start at the right place." But she began her eleventh trial at the same point again, as with the twelfth, thirteenth, and fourteenth (last).

Thus, for our measures in this problem "Control" seems to measure mainly *persistence,* with most of the variability partialled out and measured as "Varied trials."

(h) Elimination of sources of error. 42% of the subjects reported the identification and elimination of one source of error, mainly: blind-alleys (the minor variable). An unusual source of error was encountered, and met, by one subject who on her second trial unknowingly omitted the center diagonal, finished the rest of the figure, and thought that she had solved the puzzle until reminded (by the experimenter) of her omission. To meet this source of error, before beginning her next trial she marked light crosses in the five places where diagonals had to be drawn; she encountered no difficulty thereafter. This, of course, also represented "Locating a (minor) crucial aspect."

(i) Visualization. The maximum number of possible-next-moves visualized by the subjects ranged from 2 to 7, with the mean at 4.4 moves.

No other methods were revealed in the answers to the general question.

The range of the total number of methods used per subject was from 3 to 8, with the mean at 5.6 methods per subject. Again, we recognize that the mean of 5.6 blankets the fact that a given

method differed qualitatively (and also in quantitative degree) from one subject to another on the same problem.

2. *The Efficacy of the Methods Used*

The following tables show the relation between the use of a method and the number of trials involved in solution. The number of trials are presented in a two-fold classification: Superior subjects (below the median number of trials) and Inferior subjects (above the median number of trials). These small numbers of cases derived from a study mainly qualitative in design do not permit rigorous statistical treatment, but inspection of the results grouped into the 2 categories listed reveals general tendencies toward efficacy or non-efficacy of various methods.

For 3 methods where a rank-difference correlation was feasible (although no measures of reliability were obtainable), the correlation is presented.

(a) Clear formulation of the problem. All subjects seemed to formulate the problem clearly, so no measure of efficacy was possible.

(b) Preliminary survey of all aspects of the presented material. Inspection reveals that among the subjects who reported preliminary survey there were about equal numbers of superior and inferior subjects.

TABLE 10

	Number of Trials Involved in Solution	
	12 Superior Subjects (3 to 7 trials)	12 Inferior Subjects (9 trials to "failed")
Subjects making preliminary survey ($N=9$)	5	4
Subjects omitting preliminary survey ($N=15$)	7	8

And among those not reporting preliminary survey, there were also about equal numbers of superior and inferior subjects. A preliminary survey of all aspects of the stimulus-material (in the minor degree to which it appeared at all) did not seem helpful for solution of this particular problem.

(c) Analysis into major variables. The majority (3 out of 4) of those reporting the major variable were superior subjects. More inferior subjects reported the minor variable (unimportant in this problem), and about equal numbers of superior and inferior subjects reported neither variable. Awareness of the major variable seems to be related positively to successful solution.

TABLE 11

	Number of Trials Involved in Solution	
	12 Superior Subjects	12 Inferior Subjects
Detected major variable ($N=4$)	3	1
Detected minor variable ($N=9$)	3	6
Detected neither variable ($N=11$)	6	5

(d) Locating a crucial aspect of the problem. Of those who had an hypothesis which they considered crucial, there were more inferior subjects. And of those lacking such an hypothesis there were

TABLE 12

	Number of Trials Involved in Solution	
	12 Superior Subjects	12 Inferior Subjects
Hypothesis ($N=12$)	5	7
No hypothesis ($N=12$)	7	5

more superior subjects. Mere presence of an hypothesis concerning a "crucial" aspect seems, if anything at all, negatively related to successful solution for this particular problem. It is probably a matter, not of having an hypothesis but of *quality* of the hypothesis, since a poor hypothesis is not merely indifferent but actually detrimental to the speed of solution on the geometrical-figure problem.

(e) Application of past experience. No subjects reported the direct application of specific principles which could be tabulated.

(f) Varied trials. Variability correlated +.87 (P.E. = .07) with successful solution, so varied trials seem to be highly related (positively) to successful solution, because our specific measure of variability (changed starting-point) eventually led to the correct starting-point.

(g) Control. Control correlated −.61 (P.E. = .11) with successful solution, so (our measure of) control seems to be moderately related (negatively) to successful solution, because isolation and repetition of same starting-point prevented the subject from trying the only two correct starting-points.

(h) Elimination of sources of error. Of those reporting a source of error, there were more inferior subjects than superior ones. And of those not reporting a source of error, there were more superior subjects than inferior ones. In stating that elimination of a source of error was not related positively to successful solution, it must be remembered that these errors (mainly blind-alleys) could not prevent failure based almost entirely on a single factor, namely: *starting-point,* a peculiarity of this particular problem.

(i) Visualization. The number of visualized possible-next-moves correlated +.37 (P.E. = .13) with successful solution, so such visualization seems to be slightly related (positively) to successful solution.

The total number of methods used correlated −.30 (P.E. = .14) with successful solution, and seems slightly related negatively, apparently because some methods hindered solution. So the mere use of more methods does not seem to guarantee solution.

Thus, four methods show some relationship to successful solution of this problem: (*a*) Analysis into major variables, (*b*) Locating a crucial aspect, (*c*) Varied trials, and (*d*) Control. The *nature* of the relation (positive or negative) reveals that detection of the only important variable (correct starting-point), by means of varied trials rather than by persistence at the same starting-point ("Control"), leads to rapid solution of this puzzle. In summary, although the various methods of attack may be helpful in other problems, only "varied trials" and "Analysis into major variables" seem to be efficacious for solution of this problem, because the first leads to discovery of the second, and the second guarantees solution. The very construction of the figure, offering only 2 correct starting-points out of 16 possible starting-points, seems to render other methods useless.

TABLE 13

	Number of Trials Involved in Solution	
	12 Superior Subjects	12 Inferior Subjects
Detection of source of error ($N=10$)	4	6
No detection of source of error ($N=14$)	8	6

IV. DISCUSSION

IN INTERPRETING the results of this experiment, certain limitations and sources of error must be borne in mind.

1. The particular question or technique we used to determine use of a given method of attack was only one way of determining use of that method.

2. The particular question or technique employed by us often required a dichotomous answer, thus permitting only gross quantification of the use of some methods.

3. There are probably other methods of attack besides the ones for which we searched. None was revealed in our investigation.

4. Methods may be used but not reported. And, contrariwise reported use of a method does not guarantee that it was used by the subject. Our experimental technique revealed itself to be too dependent upon introspective report.

5. In interpreting the absence of efficacy of certain methods of attack, it is important to distinguish between the methods reported by untrained individuals and the possibility of application of these methods by trained problem-solvers.

6. Besides the various methods of attack used, other intellectual factors are of course partly basic to successful solution of these problems, factors not appearing as methods but contributing nevertheless to solution.

7. We used a highly select sample of subjects.

8. The three problems which we used, although different in nature, were hardly representative of typical problems.

In the following interpretations of the results of the experiment, Table 14 will be used for the first two questions, and Table 15 for the third. In Table 14 the first 3 columns show the average degree of use (based on a maximum possible use of 100%) of each method, by the group, for each of the three problems; the missing entries indicate that no measure was obtainable for certain methods in certain problems. The fourth column shows the average degree of use for three problems, and the last column shows the average degree of use for only those problems in which a measure of the method was obtainable.

1. How did individuals differ in the methods they used to solve the problems?

An obscuring factor in the presentation of the comparison of individuals with regard to use or non-use of various methods lies in the fact that the individuals differed, not so much in the kind of methods used, but rather in the extent to which each used a particular method. The individual differences in extent of use of each method for each problem have been indicated in the body of this paper; in summary it can be stated that extent of use for any one method for any one problem appeared to be distributed normally, considering the limitations of measurement (gross measurement, dichotomous answers, etc.). If we credit an individual with use of a method, regardless of degree of use, the differences in the number of methods used on each problem only approximate a normal curve; since each of the three distributions represents, not a single variable, but rather a total use of any or all of 9 variables (methods), we would not necessarily expect normal distributions.

a. In the induction problem, where only 3 methods were used, every subject used either 2 or 3 methods, with the mean at 2.4 methods per subject.

b. In the deduction problem, where 8 methods were used, subject used from 3 to 7 methods, with the mean at 4.7 methods per subject.

c. In the geometrical-figure problem, where 8 methods were used, subject used from 3 to 8 methods, with the mean at 5.6 methods per subject.

2. To what extent were the methods used in different problems?

In Table 14, the fourth column of fig-

TABLE 14

AVERAGE EXTENT OF USE OF METHODS
(All figures based on maximum possible or 100% use)

Method	Induction Problem	Deduction Problem	Geometrical Figure	For Three Problems*	For Relevant Problems*
Clear formulation problem	40%	100%	100%	80%	80%
Preliminary survey	100%	45%[b]	38%	61%	61%
Analysis into major variables	70%[a]	Same as "crucial aspect" or "varied trials"	17%	29%	43.5%
Locating a crucial aspect	Not relevant?	74%[c]	50%	41%	62%
Application of past experience	0%	12%	0%	4%	12%
Varied trials	Same as "major variables"?	28%	56%	28%	42%
Control	Same as "major variables"?	44%	51%	32%	47.5%
Elimination of sources of error	Not relevant?	51%[d]	42%	31%	46.5%
Visualization	Not relevant?	84%	15%[e]	33%	49.5%
Total number of methods used on problem	3	8	8		
Mean-number of methods used per subject	2.4	4.7	5.6		
Range of number of methods used by subjects on each problem	2–3	3–7	3–8		

[a] Based on maximum of 8 major variables.
[b] Based on maximum of 4 readings (highest number by any subject).
[c] Based on maximum of 9 crucial aspects.
[d] Based on maximum of 8 sources of error.
[e] Based on maximum of 29 possible visualized next-moves.
* These figures are intended as suggestive only, and have not been corrected for small differences in the number of subjects in each of the three groups.

ures shows that some methods seem hardly to have been used at all. For example, "Application of past experience" seems to be almost completely absent (4%), and another 5 methods show use of only 28% to 33%. Inspection reveals, however, that "Application of past experience" was reported (by one or more subjects) on only one of the three problems; when the figure is based on the problem where the method was used, we see that 12% of the subjects reported its use. Further inspection reveals that the problem was a human relations situation (death of two men), in which past experience would more likely be applicable, as compared with the two problems of a more perceptual nature. Even in these, however, it is the author's impression that finer techniques of investigation might

have revealed subtle use of the methods in question.

For the 5 methods showing use of only 28% to 33%, inspection reveals that each method was measurable on only two problems; when the figures are based on the two problems where used, the range becomes 42% to 49.5%.

Why were measures unobtainable for certain methods on certain problems? Two reasons are suggested.

a. For a given problem, certain presumably different methods appeared (to both the experimenter and the subjects) to carry the same meaning. On the induction problem, for example, "Varied trials" and "Control" (two aspects of trial-and-error) consisted of applying generalizations to the groups of letters; this method was measured as "Analysis into major variables." On the deduction problem, "Analysis into major variables" could be considered as looking for clues, measured as "Locating a crucial aspect," or as questioning each of the three possible causes (murder, accident, suicide) in relation to the death of each man, measured as "Varied trials." It matters little whether the subject's behavior is classified under one or the other of two headings; the important thing is that a distinction should be made between "methods" different in terminology only and methods different in actual behavior of the subjects.

b. A similar point is that, on a given problem certain "methods" seem either meaningless or irrelevant. On the induction problem, for example, the experimenter was unaware of any "crucial aspects" among the groups of letters; if the common characteristics are crucial, then this method is identical with "Analysis into major variables." On the same problem the experimenter was unaware of sources of error in the stimulus-material; and "Visualization" seemed to be irrelevant, in the sense that all the material was visual and seemed to require no additional visualization (imagery). The introspective reports confirmed the absence of such "methods."

We conclude, therefore, that although there is wide variation in the extent to which different methods are used, extent of use is partly a function of relevancy of certain methods to certain problems. In Table 14, for example, inspection reveals that only a few methods (3) are used on the induction problem, whereas most of the methods (8) are used on the other two problems; and the average use per subject follows the total number of methods used. Since the individuals do not change, we suggest that the number of methods used in different problem-situations depends upon the kind of problem involved: (*a*) some problems "permit" use of a given method (for example, application of past experience) while other problems do not; (*b*) in certain situations a given "method of attack" becomes identical in meaning with a differently worded method of attack; and (*c*) in certain situations a given "method of attack" is a term rather than a form of human behavior.

3. To what extent were the different methods efficacious?

Table 15 summarizes the degree of efficacy for each method on each problem.

a. Clear formulation of the problem showed no relationship to solution for one problem and was of unknown efficacy for the other two problems. It is suggested that the task may have been quite obvious in our problems, thereby permitting no individual differences in understanding of the requirements of the problem.

b. Preliminary survey of all aspects of the presented material was of slight help in the deduction problem, of unknown efficacy in the induction problem, and of no efficacy in the geometrical-figure problem. It is suggested that success in the latter two problems requires considerable trial-and-error rather than global survey.

c. Analysis into major variables was moderately helpful in the induction problem, slightly helpful in the geometrical-figure problem, and unmeasura-

TABLE 15

EFFICACY OF METHODS OF ATTACK

Correlations indicated when obtainable. Degree of relationship ("Zero" or "slight") indicated where correlations could not be derived. Relationships not obtainable for: (*a*) methods not measurable and (*b*) methods showing no substantial individual differences.

Method	Induction Problem	Deduction Problem	Geometrical Figure
Clear formulation of problem	−.10	Used by all: efficacy not obtainable	Used by all: efficacy not obtainable
Preliminary survey	Used by all: efficacy not obtainable	+.22	Zero
Analysis into major variables	+.51 (P.E. = .11)		Slightly positive
Locating a crucial aspect		+.39 (P.E. = .15)	Slightly negative
Application of past experience	Used by none	Used by only three: efficacy not obtainable	Used by none
Varied trials		Slightly positive	+.87 (P.E. = .07)
Control		Slightly positive	−.61 (P.E. = .11)
Elimination of sources of error		+.60 (P.E. = .12)	Slightly negative
Visualization		Zero	+.37 (P.E. = .13)
Total number of methods used	Number of methods insufficient to permit correlation	+.48 (P.E. = .14)	−.30 (P.E. = .14)

ble in the deduction problem. It is suggested that this method tends to be helpful because it may expose a variable fundamental to solution (along with unimportant variables).

d. Locating a crucial aspect was slightly helpful in the deduction problem, slightly disadvantageous in the geometrical-figure problem, and unmeasurable in the induction problem. It is suggested that the potential efficacy of the search for crucial aspects is contingent upon the time or trials available to the problem-solver: with unlimited time (as in the deduction problem), the method tends to be helpful; with limited time or trials (as in the geometrical-figure problem), the method may obstruct solution, especially if one hypothesis be perseverated (Maier's "false direction").

e. Application of past experience was not reported by our subjects, so no measure of efficacy was obtainable. On the geometrical-figure problem, however, the work-sheets indicated a tendency to start at the same point (usually the upper-left) on consecutive trials, thus making solution impossible. It is suggested that previous practice in tracing figures acts as negative transference, because most tracing-figures can be solved from any starting-point, so long as care is exercised to avoid blind-alleys.

f. Varied trials was slightly helpful

in the deduction problem, paramount in the geometrical-figure problem, and unmeasurable in the induction problem. Recognizing, however, that in the geometrical-figure problem our only measure of variation was variation of starting point, and that correct starting-point is the only important factor for this particular problem, it is suggested that the efficacy of variation is dependent upon its potentiality of exposing the major variable or crucial aspect of a problem.

g. Control was slightly helpful in the deduction problem, detrimental in the geometrical-figure problem, and unmeasurable in the induction problem. It is suggested that the efficacy of "isolation and repetition" of variables felt to be important by the subject, or the efficacy of aspects felt to be crucial, is dependent upon the subject's luck or skill in testing the important variable or aspect.

h. Elimination of sources of error was moderately helpful in the deduction problem, slightly disadvantageous in the geometrical-figure problem, and unmeasurable in the induction problem. It is suggested that the efficacy of this method is dependent upon its potentiality of exposing significant features of the situation.

i. Visualization showed a small positive correlation with successful solution in the geometrical-figure problem, no effect in the deduction problem, and was unmeasurable in the induction problem. Visualization could be of no help on the geometrical-figure problem unless a subject could visualize all 29 moves, because 2 of the 16 starting-points practically guarantee solution, whereas 14 starting-points make solution impossible with or without visualization. The small positive correlation in the geometrical-figure problem may indicate the typical positive correlation of abilities rather than a causal relationship. It is suggested that, as with several other methods already discussed, the efficacy of visualization (and other imagery) is dependent upon the type of problem.

V. SUMMARY AND CONCLUSIONS

THE LITERATURE on reasoning reveals various methods, techniques and auxiliary aids used by subjects during problem-solving. Nine of such known "methods of attack" were selected for investigation in this experiment.

1. Clear formulation of the problem.
2. Preliminary survey of all aspects of the presented material.
3. Analysis into major variables.
4. Locating a crucial aspect of the problem.
5. Application of past experience.
6. Varied trials.
7. Control.
8. Elimination of sources of error.
9. Visualization.

Three different problems were selected to be given to the subjects serving in this experiment. The induction problem was a short concept-formation test (identification of similarities and differences in letter-grouping series) devised by Professor Thurstone; 19 items were used. The deduction problem, chosen from one of Professor Thurstone's reasoning tests, presented a paragraph of clues to the death of two men; the subject is to ascertain the cause of the death of each man. The general problem was a geometrical-figure problem of a maze-tracing variety. The problems were administered individually to a group of 25 undergraduate students, with an unlimited time-allowance. A number of specific questions were asked of the subjects during and after the work-period, and objective records of the work-sheets were kept, designed to determine the use of the various methods of attack selected for investigation.

Our problem was to determine the nature and efficacy of the methods of attack used on different problems. The three main questions, and the results, were as follows.

1. How did individuals differ in the methods they used to solve the problems?

a. If we credit an individual with use of a method, disregarding *degree* of use, the number of methods used was not distributed normally for either all three problems combined or for any one problem. It appears to us that normal distributions would not necessarily be expected, considering that each distribution represents not a single variable or continuum, but rather a total use of any or all of nine variables (methods).

b. In the induction problem, every subject used either two or three methods, with the mean at 2.4 methods. In the deduction problem, subjects used from 3 to 7 methods, with the mean at 4.7 methods. In the geometrical-figure problem, subjects used from 3 to 8 methods, with the mean at 5.6 methods.

c. Individual differences in degree of use of a particular method on a given problem appeared to be distributed normally, considering the limitations of measurement.

2. To what extent were the methods used in different problems?

There was wide variation in the extent to which different methods were used, certain methods being used on one problem, and other methods on a different problem. Two methods were used on all three problems, six methods were used on two problems, and one method was used on only one of the three problems. Examination of the tasks required in the three problems suggests that, for a specific problem: (a) certain presumably-different methods appeared to carry the

same meaning, and (b) certain so-called "methods" appeared to be meaningless or irrelevant.

3. To what extent were the different methods efficacious?

a. For a given problem some methods were efficacious, some were indifferent, and some were detrimental to solution. The latter case seemed to occur whenever a particular method served to prevent or postpone discovery of the variable or aspect fundamental to solution of the problem.

b. Because no method was used on all three problems (except "Clear formulation," used by almost everyone, and thus of unmeasurable efficacy), efficacy of a given method was demonstrable for only one or two of the three problems. For the latter case (two problems), two methods were efficacious on both problems, two methods were efficacious on one problem and "indifferent" (zero correlation) on the other, and three methods were efficacious on one problem and "detrimental" (negative correlation) on the other.

In interpreting the results, however, certain limitations and sources of error were recognized:

1. The particular question or technique used to determine use of a given method was only one way of determining use of that method.
2. The particular question or technique employed by us often required a dichotomous answer, thus permitting only gross quantification of the use of methods.
3. There were probably other methods used which escaped our tools of search.
4. Some of the nine methods may have been used but not reported and not demonstrated in the work-sheets.
5. Various uncontrolled intellectual factors were also contributing to successful solution.
6. We used a highly select sample of subjects.
7. The three problems which we used, although different in nature, were hardly representative of typical problems.

Four general conclusions are suggested as a result of this experiment.

1. The extent of use of a particular method of attack varies with different problems and is dependent upon the kind of task involved.

2. On a particular problem, certain presumably different methods may be identical.

3. On a particular problem, certain so-called "methods" may be meaningless: mere terms having no meaningful reference to possible behavior.

4. The potential efficacy of a particular method of attack depends upon the kind of problem involved.

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